Laws Of Motion

CHAPTER-5

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(5) Laws of motion

Concept of force

Effects of force

Inertia → (Types)

(Applications)

First law of motion

Linear momentum

$\vec{p} = m\vec{v}$

Third law of motion

Action = − Reaction

Force

Force

Second law of motion → Impulse → Applications

$\Delta p = F \cdot \Delta t$

$\vec{F} = d\vec{p}/dt$

$\vec{F} = m\vec{a}$

$\vec{F}_{ext} = m\vec{a}$

Conservation of linear momentum → (Applications)

$\vec{F}_{net} = 0$

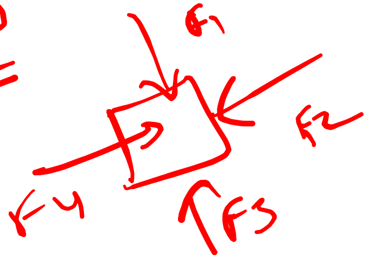
$d\vec{p}/dt = 0 \Rightarrow \vec{p} = \text{constant}$

Rest

uniform st. line motion

Equilibrium of Concurrent forces

$\vec{F}_{net} = 0$



Weight of a man in a lift.

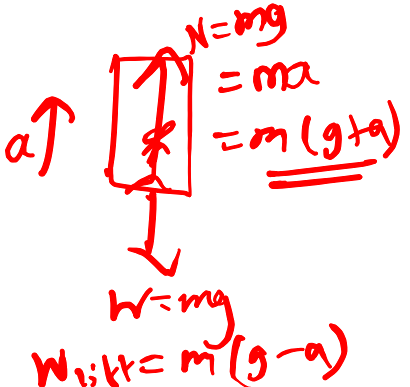
$W = mg$

□ ↑ a

$W = m(g+a)$

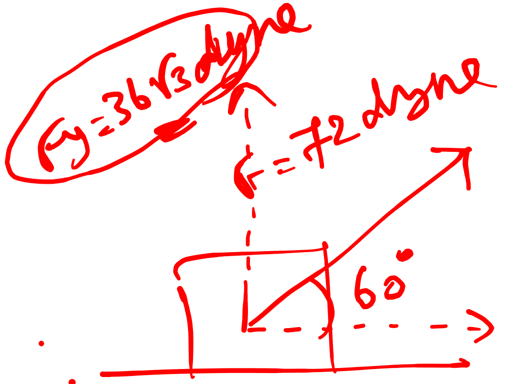
□ ↓ a

$W = m(g-a)$



(1) A force of 72 dyne is inclined to the horizontal at an angle of 60°. Find the acceleration in a mass of 9g, which moves in a horizontal direction.

$$\vec{F} = F_x \hat{i} + F_y \hat{j}$$



$$F_x = F\cos 60^\circ$$
$$= 72 \cos 60^\circ$$
$$= 36 \text{ dyne}$$

$$m = 9g$$

$$F_x = m a_x$$

$$a_x = F_x / m = \frac{\cancel{36} \text{ dyne}}{\cancel{9}\, g} = 4 \text{ dyne } g^{-1}$$

$$= 4\, \cancel{g}\, cm\, s^{-2} . \cancel{g^{-1}}$$

$$\boxed{a_x = 4\, cm\, s^{-2}}$$

(2) Calculate the impulse necessary to stop a 1500 kg car travelling at 90 km/h.

Sol:

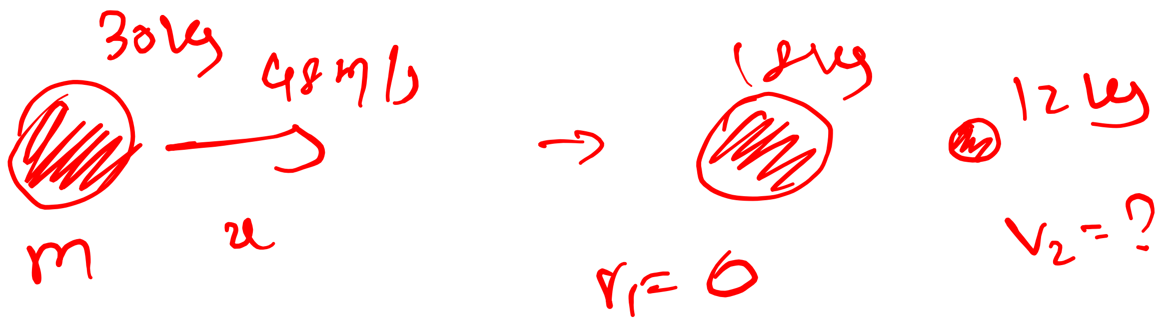
$u = 90\ km/h = \cancel{90}^{5} \times \frac{5}{\cancel{18}} = 25\ m/s$

$v = 0$; $m = 1500\ kg$.

$Impulse = F \cdot t = \Delta p = mv - mu = 1500(0) - 1500(25)$

$$\boxed{I = -37500\ Ns}$$

③ A 30 kg shell is flying at 48 m/s. When it explodes, its one part of 18 kg stops, while the remaining part flies on. Find the velocity of the latter.



$$TMBR = TMAR$$

$$Mu = m_1 v_1 + m_2 v_2$$

$$30(48) = 18(0) + 12(v_2)$$

$$\therefore v_2 = \frac{1440}{12}$$

$$v_2 = 120 \text{ m/s}$$

(4) Two masses m_1 and m_2 are connected at the two ends of a light inextensible string that passes over a frictionless pulley. Find the acceleration of the masses and the tension in the string, when the masses are released. $(m_2 > m_1)$.

$$\boxed{F_{exes} = ma}$$

Sol: for mass m_2 ;

$$m_2 g - T = m_2 a \rightarrow (1)$$

$$T - m_1 g = m_1 a \rightarrow (2)$$

$$m_2 g - m_1 g = m_1 a + m_2 a$$

$$(m_2 - m_1) g = (m_1 + m_2) a$$

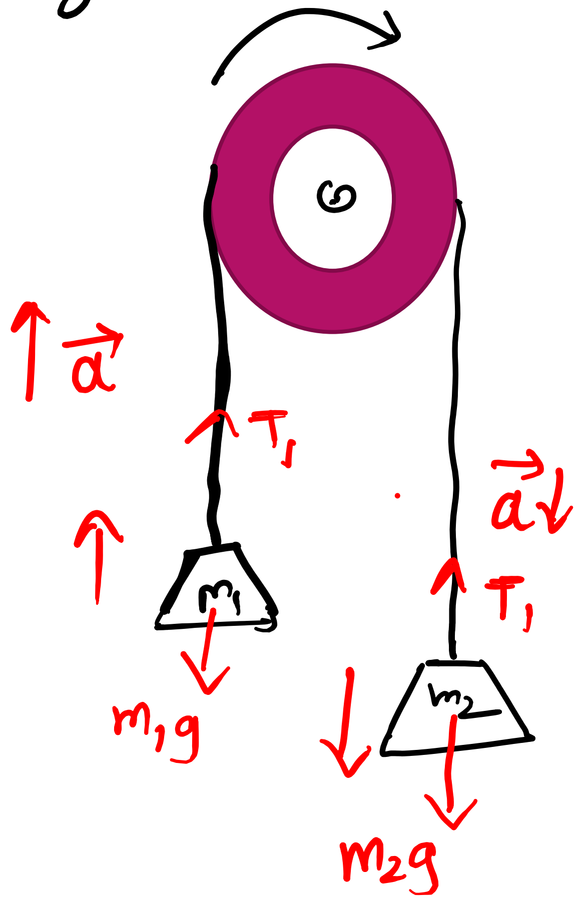
$$\boxed{a = \frac{(m_2 - m_1) g}{m_1 + m_2}}$$

$$T = m_1 a + m_1 g$$

$$T = m_1 (a + g)$$

$$T = m_1 \left(\frac{(m_2 - m_1) g}{m_1 + m_2} + g \right)$$

$$T = m_1 \left(\frac{m_2 g - \cancel{m_1 g} + \cancel{m_1 g} + m_2 g}{m_1 + m_2} \right) \Rightarrow T = \frac{2 m_1 m_2 g}{m_1 + m_2}$$



(5) A hydrogen gas filled balloon having a mass of 25g is released up in the air. As the balloon ascends, the gas starts leaving from it with a uniform velocity of 12 cm s^{-1} and as a result, the balloon shrinks completely in 5s. Find the average force acting on the balloon.

Sol:-

$F = ma$

$\downarrow$

$F \propto \frac{d\vec{p}}{dt}$

$F = k\frac{dp}{dt}$

$F = \frac{dp}{dt} = \frac{d}{dt}(mv)$

$\vec{F} = m\frac{dv}{dt} = ma$

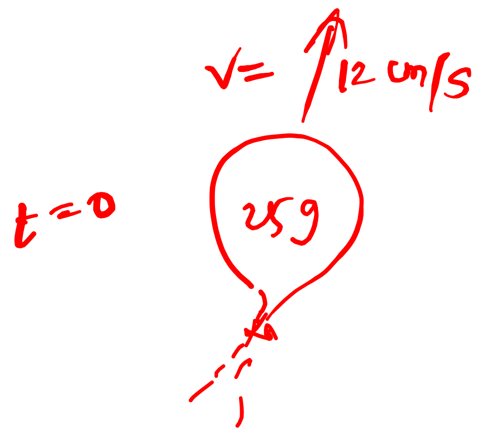
$F = v\frac{dm}{dt}$

$F = 12\ cm/s \times \frac{\cancel{25}^{5}}{\cancel{5}}\ g/s$

$F = 60\ g\ cm\ s^{-2}$

$\boxed{F = 60\ dyne}$

$t = 5s$



⑥ A metallic plate of mass 100g is kept floating in air horizontally by firing 10 bullets s^{-1} vertically upwards, which rebound from the plate with the same speed in opposite direction. If the mass of a bullet is 25g, find the speed at which they are fired at the plate. Given that $g = 10\ m/s^2$.

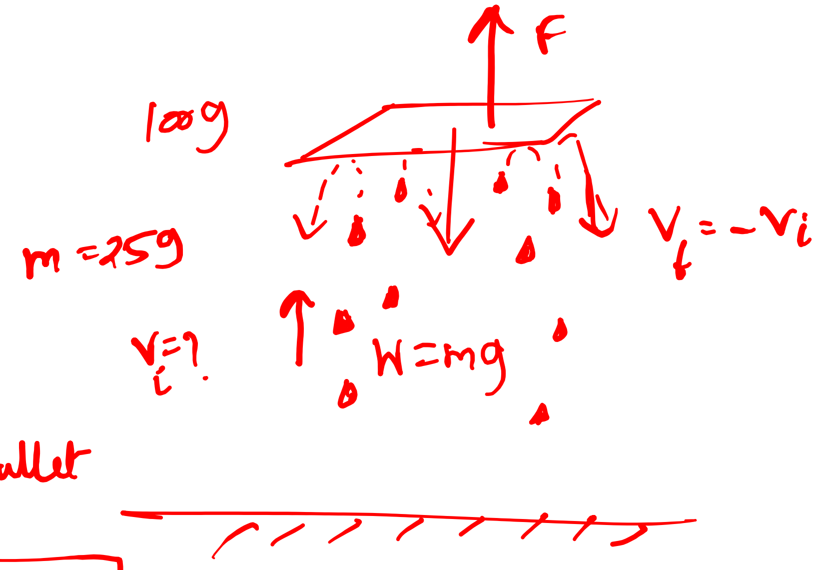
$F = W = mg \rightarrow ①$

$F_{bullet} = -F = -mg$

$F_{bullet} = \frac{\Delta p}{\Delta t} = \frac{mv - mu}{\Delta t} = \frac{m(v - (-v))}{\Delta t} = \frac{2mv}{\Delta t} = \underline{\underline{2mv}}$

↳ by one bullet

$\Delta t = 1s$



$\therefore F_{10 bullets} = 2(10)mV = 20mV \rightarrow ②$

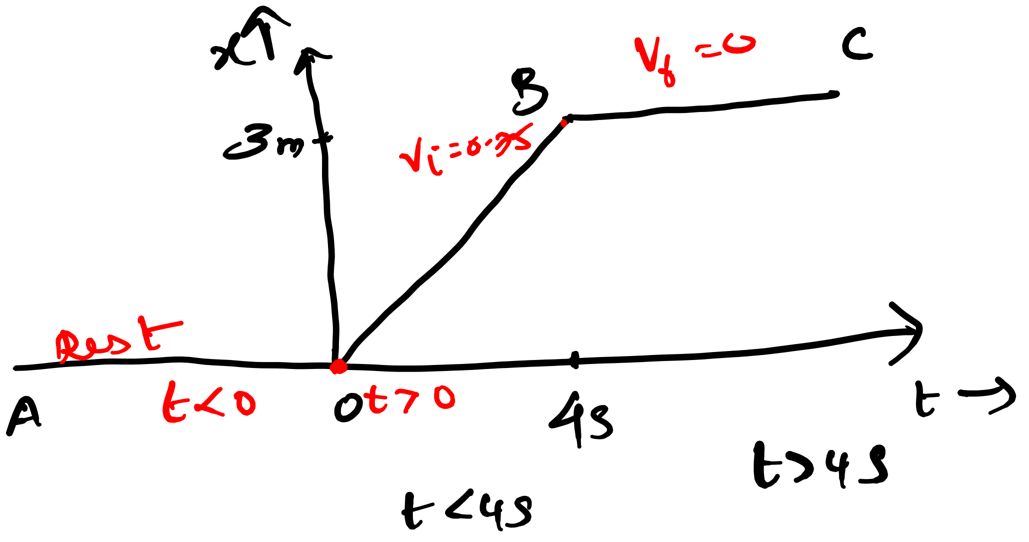
$\boxed{\cancel{\frac{10}{20}} = 0.5\ m s^{-1}}$

$\cancel{20 \dots}(10)$

$20\,(25\,\cancel{g}) \times V = 100\,\cancel{g}\,(10)$

$V = \frac{1000}{500} = \underline{\underline{2\ m/s}}$

(7) Figure given below shows the position - time graph of a particle of mass 4 kg. Find the impulse at $\underline{t=0}$ and $\underline{t=4s}$. The motion may be considered one dimensional.



Sol: Impulse $= \Delta p = \Delta P_f - \Delta P_i$

When $t<0$; $V_i = 0$;

$t>0$; $V_f = 0.75 \, m/s$

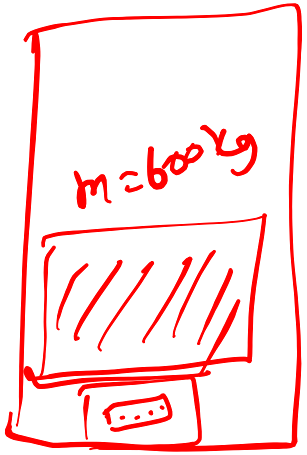
$\Delta p = 3 \, kg\,ms^{-1}$ at $t=0$

When $t = 4$:- $V_i = 0.75 \, m/s$ $V_f = 0$

$\Delta p = m(V_f - V_i) = 4(0 - 0.75)$

$\Delta p = -3 \, kg\,ms^{-1}$

⑧ A heavy load of mass 600 kg is placed on the weighing machine lying in a lift. What will be the reading of a weighing machine, when the lift is (a) at rest (b) moving upwards with the acceleration of $2.2\ m/s^2$, (c) moving downwards with an acceleration of $2.8\ m/s^2$ and (d) falling freely due to the rupture of the cable ?



(a) $W = mg = 600 \times 9.8 \Rightarrow 5880\ N$

(b) $W_{up} = m(g+a) = 600(9.8+2.2)) = 600 \times 12$
$= 7200\ N$

(c) $W_{down} = m(g-a) = 600(9.8-2.8) = 600 \times 7$
$= 4200\ N$

(d) falling freely $= m(g-a) = m(g-g) = 0$

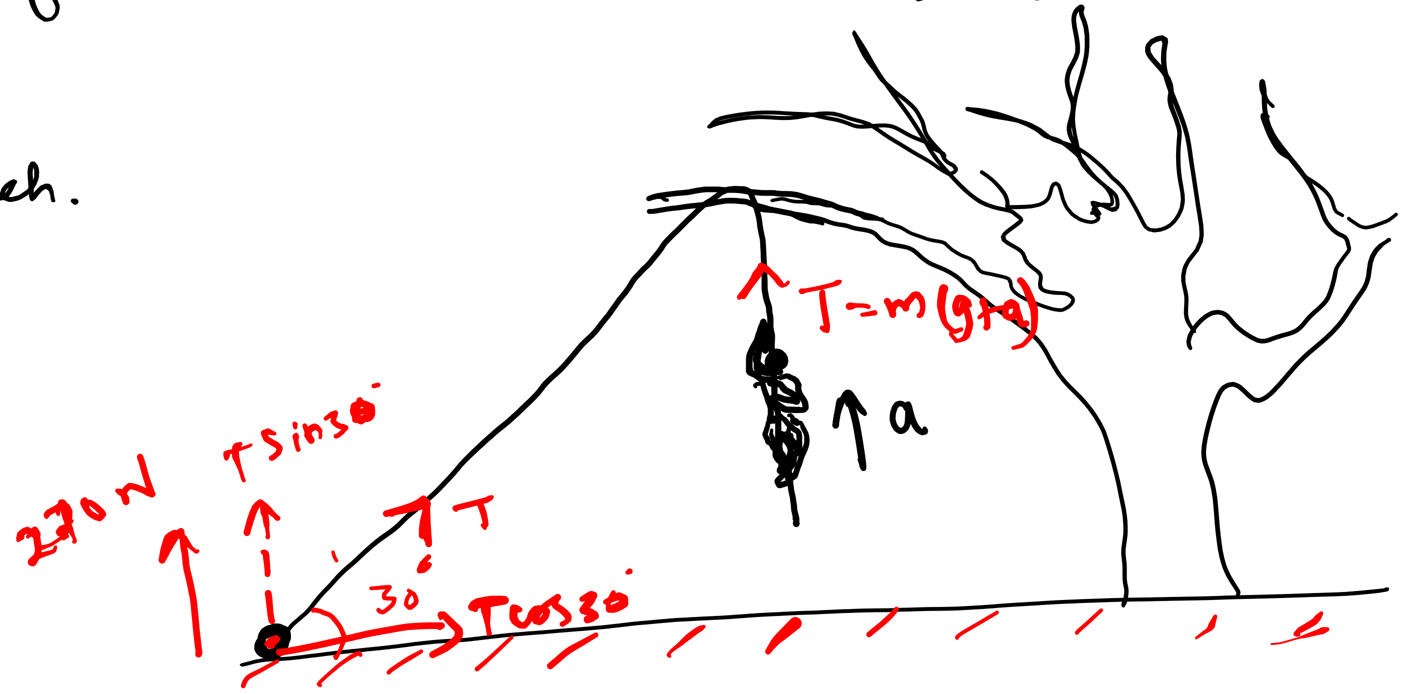
(9) Figure shows a light rope fixed at one end to a clamp on the ground and its other end passing over the branch of a tree and hanging on the other side of it. The rope makes an angle of 30° with the ground.

A boy weighing 45 kg starts climbing up the rope. Find the maximum acceleration with which the boy can climb safely, if the clamp comes out of the ground, when a force of 270 N acts on it vertically upwards. Assume that there is no friction between the rope and the tree branch.

Take $g = 10 \text{ m/s}^2$.

If $T\sin\theta = 270$

$m(g+a)\sin 30^\circ = 270$



$$m(g+a)\sin 30^\circ = 270$$

$$45(10+a)\tfrac{1}{2} = 270$$

$$45(10+a) = 540$$

$$10+a = \frac{\cancel{540}\ 12}{\cancel{45}}$$

$$a = 12 - 10 = \underline{\underline{2\ m/s}}$$